

Logarithmic Functions

- 1) $\log a \cdot b = \log a + \log b$
 - 2) $\log \frac{a}{b} = \log a - \log b$
 - 3) $\log a^n = n \log a$
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① $y = \ln x$
 $y' = \frac{1}{x}$

Examples

1) $y = \ln(x^2 - x + 5)$
 $y' = \frac{(2x - 1)}{(x^2 - x + 5)}$

2) $y = \ln(\sin x + \cos x)$
 $y' = \frac{(\cos x - \sin x)}{(\sin x + \cos x)}$

3) $y^2 + \ln xy = x^2$
 $y^2 + (\ln x + \ln y) = x^2$
 $2yy' + \frac{1}{x} + \frac{1}{y} \cdot y' = 2x$
 $y'(2y + \frac{1}{y}) = 2x - \frac{1}{x}$
 $\therefore y' = \frac{2x - \frac{1}{x}}{2y + \frac{1}{y}} = \frac{y(2x^2 - 1)}{x(2y^2 + 1)}$

4) $\ln(x^2 + y^2) = x + 2y$
 $\frac{2x + 2yy'}{x^2 + y^2} = 1 + 2y'$
 $2x + 2yy' = (x^2 + y^2)(1 + 2y')$
 $y'(2y - 2x^2 - 2y^2) = x^2 + y^2 - 2x$
 $\therefore y' = \frac{x^2 + y^2 - 2x}{2y - 2x^2 - 2y^2}$

5) $y = \ln(x^2 + y^2) + \ln(xy)$

sol

$$6) y = \ln 2x$$

$$y' = \ln 2 + \ln x = 0 + \frac{1}{x} = \frac{1}{x}$$

$$7) y = \ln x^2 = 2 \ln x$$

$$y' = 2 \ln x = \frac{2}{x}$$

$$8) y = 2 \ln(3x^2 - 1)$$

$$y' = 2 \frac{6x}{3x^2 - 1} = \frac{12x}{3x^2 - 1}$$

$$9) y = \ln(1 - 2x)^3$$

$$y' = 3 \frac{-2}{1 - 2x} = \frac{-6}{1 - 2x}$$

$$10) y = \ln[(\sin 2x)(\sqrt{x^2 + 1})]$$

$$y = \ln[\sin 2x] + \ln[\sqrt{x^2 + 1}]$$

$$y = \ln[\sin 2x] + \ln(x^2 + 1)^{1/2}$$

$$y = \ln[\sin 2x] + \frac{1}{2} \ln(x^2 + 1)$$

$$y' = \frac{2 \cos 2x}{\sin 2x} + \frac{1}{2} \left(\frac{2x}{x^2 + 1} \right)$$

$$y' = 2 \cot 2x + \frac{x}{x^2 + 1}$$

$$11) y = \ln(2x^3 - x)^2 = 2 \ln(2x^3 - x)$$

$$y' = 2 \frac{6x^2 - 1}{2x^3 - x}$$

$$12) y = \ln(\cos x^2)$$

$$y' = \frac{-2x \sin x^2}{\cos x^2} = -2x \tan x^2$$

$$13) y = x \ln^3 x = x(\ln x)^3$$

$$y' = x \frac{3(\ln x)^2}{x} + (\ln x)^3 \cdot (1)$$

$$= 3(\ln x)^2 + (\ln x)^3$$

$$= [\ln(x)]^2 [3 + \ln x]$$

$$14) 3 \ln xy + \sin y = x^2$$

$$3(\ln x + \ln y) + \sin y = x^2$$

$$3\left(\frac{1}{x} + \frac{1}{y} \cdot y'\right) + \cos y \cdot y' = 2x$$

~~$$3\left(\frac{1}{x} + \frac{1}{y} \cdot y'\right)$$~~

$$\frac{3}{y} \cdot y' + \cos y \cdot y' = 2x - \frac{3}{x}$$

$$y' \left(\frac{3}{y} + \cos y \right) = 2x - \frac{3}{x}$$

$$y' = \frac{2x - \frac{3}{x}}{\frac{3}{y} + \cos y} = \frac{2x^2y - 3y}{3x + xy \cos y}$$

$$15) y = \ln \frac{x}{3x-4}$$

$$y' = \ln x - \ln(3x-4)$$

$$y' = \frac{1}{x} - \frac{3}{3x-4}$$

$$y' = \frac{3x-4-3x}{x(3x-4)}$$

$$y' = \frac{-4}{x(3x-4)}$$

$$16) y = (\sin x)^x$$

$$\ln y = \ln (\sin x)^x$$

$$= x \ln \sin x$$

بالطرفين ln

$$\frac{1}{y} \cdot y' = x \frac{\cos x}{\sin x} + \ln \sin x \cdot (1)$$

$$y' = y(x \cot x + \ln \sin x)$$

بالتعويض عن قيمة y :-

$$y' = (\sin x)^x (x \cot x + \ln \sin x)$$