Integration   التكامل

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

Examples: ① $\int x^2 \, dx = \frac{x^3}{3} + c$

② $\int x^{1/2} \, dx = \frac{x^{1/2+1}}{\frac{1}{2}+1} + c = \frac{2}{3} x^{3/2} + c$

③ $\int \frac{dx}{x^2} = \int x^{-2} \, dx = \frac{x^{-1}}{-1} + c = -\frac{1}{x} + c$

④ $\int (3x^2 + 2x + 1) \, dx = \frac{3x^3}{3} + \frac{2x^2}{2} + x + c = x^3 + x^2 + x + c$

⑤ $\int \frac{x^3 + x^2}{x^5} \cdot dx = \int \left(\frac{1}{x^2} + \frac{1}{x^3}\right) dx = \int (x^{-2} + x^{-3}) \, dx$

$$= -x^{-1} - \frac{x^{-2}}{2} + c = -\frac{1}{x} - \frac{1}{2x^2} + c$$

⑥ $\int (1+x^2)^2 \cdot dx = \int (1 + 2x^2 + x^4) \cdot dx$

$$y = x + \frac{2x^3}{3} + \frac{x^5}{5} + c$$

⑦ $\int \sqrt[3]{x^2} \cdot dx = \int x^{2/3} \cdot dx = \frac{3x^{5/3}}{5} + c$

⑧ $\int (3x^2 - 2x + 1)(x^3 - x^2 + x - 5)^9 \cdot dx$

$$y = \frac{(x^3 - x^2 + x - 5)^{10}}{10} + c$$

Examples,

1- $\int (2x + 3) \cdot dx$

2- $\int (x-1)^{243} \cdot dx$

3- $\int \sqrt{1-x} \cdot dx$

4- $\int (2-t)^{2/3} \cdot dt$

5- $\int (1+x^3)^2 \cdot dx = \int (1 + 2x^3 + x^9) \cdot dx \Rightarrow y = x + \frac{2}{2}\frac{x^4}{4} + \frac{x^{10}}{10} = \boxed{x + \frac{x^4}{2} + \frac{x^{10}}{10}}$

6- $\int 3x^2 \sqrt{1+x^3} \cdot dx$

7- $\int \sqrt{2+5y} \cdot dy$

8- $\int t^2 (1+2t^3)^{-(\frac{2}{3})} \cdot dt = \frac{1}{6} \int 6t^2 (1+2t^3)^{-\frac{2}{3}} \cdot dt = \frac{1}{6} \cdot \frac{(1+2t^3)^{1/3}}{1/3} + c$

$$= \frac{1}{2}(1+2t^3)^{1/3} + c$$

9- $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \cdot dx = \int (x^{1/2} + x^{-\frac{1}{2}}) \cdot dx = \frac{2}{3} x^{3/2} + 2x^{1/2} + c$

10- $\int (x^2 - \sqrt{x}) \cdot dx = \int (x^2 - x^{1/2}) \cdot dx = \frac{x^3}{3} - \frac{2}{3} x^{3/2} + c$

①

11- $\int (2-7t)^{2/3} \cdot dt$

12- $\int \dfrac{1}{\sqrt{1-x}} \cdot dx$

13- $\int (3x-1)^{234} \cdot dx = \dfrac{1}{3} \int 3(3x-1)^{234} \cdot dx = \dfrac{1}{3} \dfrac{(3x-1)^{235}}{235} + C$

14- $\int (1-x^3)^2 \cdot 3x^2 \cdot dx = -\int -3x^2 (1-x^3)^2 \cdot dx = \dfrac{-(1-x^3)^3}{3} + C$

15- $\int \dfrac{3x^2}{\sqrt{1+x^3}} \cdot dx = \int 3x^2 (1+x^3)^{-\frac{1}{2}} \cdot dx = \dfrac{(1+x^3)^{1/2}}{1/2} + C$

16- $\int \dfrac{dx}{(3x+2)^2} \cdot dx$

17- $\int x\sqrt{(2x^2+1)} \cdot dx = \int x(2x^2+1)^{1/2} \cdot dx = \dfrac{1}{4} \int 4x(2x^2+1)^{1/2} \cdot dx$

$= \dfrac{1}{4} \cdot \dfrac{(2x^2+1)^{3/2}}{3/2} + C$

18- $\int \dfrac{y\,dy}{\sqrt{2y^2+1}} = \int y(2y^2+1)^{-\frac{1}{2}} \cdot dy = \dfrac{1}{4} \int 4y(2y^2+1)^{-\frac{1}{2}} \cdot dy$

$= \dfrac{1}{4} \cdot \dfrac{(2y^2+1)^{1/2}}{1/2} + C$

19- $\int \dfrac{(z+1)\,dz}{\sqrt{z^2+2z+2}} = \dfrac{1}{2}\int 2(z+1)(z^2+2z+2)^{-\frac{1}{2}} \cdot dz$

$= \dfrac{1}{2} \dfrac{(z^2+2z+2)^{1/2}}{1/2} + C$

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كذلك في الدوال المثلثة

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1- $\int \cos x \, dx = \sin x + C$ $\rightarrow$ $y = \sin x$, $y' = \cos x$

2- $\int \sin x \, dx = -\cos x + C$ $\rightarrow$ $y = \cos x$, $y' = -\sin x$

3- $\int \sec x \tan x \cdot dx = \sec x + C$ $\rightarrow$ $y = \sec x$, $y' = \sec x \tan x$

4- $\int \sec^2 x \, dx = \tan x + C$ $\rightarrow$ $y = \tan x$, $y' = \sec^2 x$

5- $\int \csc^2 x \cdot dx = -\cot x + C$ $\rightarrow$ $y = \cot x$, $y' = -\csc^2 x$

6- $\int \csc x \cot x \cdot dx = -\csc x + C$ $\rightarrow$ $y = \csc x$, $y' = -\csc x \cot x$

②